OCR
RECOGNISING ACHIEVEMENT

Thursday 21 June 2012 – Afternoon

A2 GCE MATHEMATICS (MEI)

4756 Further Methods for Advanced Mathematics (FP2)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4756
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

4715740612

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions in Section A and **one** question from Section B.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (54 marks)

Answer all the questions

1 **(a)** **(i)** Differentiate the equation $\sin y = x$ with respect to x, and hence show that the derivative of $\arcsin x$ is $\dfrac{1}{\sqrt{1-x^2}}$. **[4]**

(ii) Evaluate the following integrals, giving your answers in exact form.

(A) $\displaystyle\int_{-1}^{1} \frac{1}{\sqrt{2-x^2}}\,\mathrm{d}x$ **[3]**

(B) $\displaystyle\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{\sqrt{1-2x^2}}\,\mathrm{d}x$ **[4]**

(b) A curve has polar equation $r = \tan\theta$, $0 \leqslant \theta < \frac{1}{2}\pi$. The points on the curve have cartesian coordinates (x, y). A sketch of the curve is given in Fig. 1.

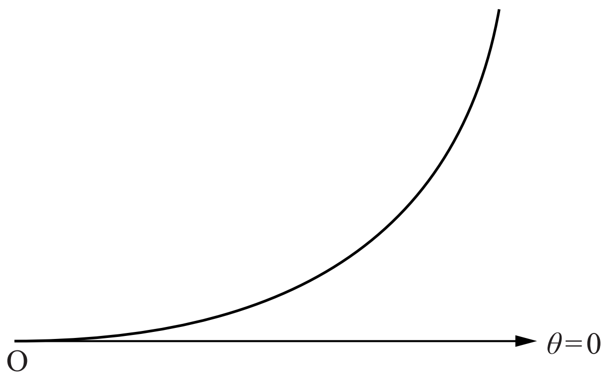

Fig. 1

Show that $x = \sin\theta$ and that $r^2 = \dfrac{x^2}{1-x^2}$.

Hence show that the cartesian equation of the curve is

$$y = \frac{x^2}{\sqrt{1-x^2}}.$$

Give the cartesian equation of the asymptote of the curve. **[7]**

2 **(a)** **(i)** Given that $z = \cos\theta + \mathrm{j}\sin\theta$, express $z^n + \dfrac{1}{z^n}$ and $z^n - \dfrac{1}{z^n}$ in simplified trigonometric form. **[2]**

(ii) Beginning with an expression for $\left(z + \dfrac{1}{z}\right)^4$, find the constants A, B, C in the identity

$$\cos^4\theta \equiv A + B\cos 2\theta + C\cos 4\theta.$$ **[4]**

(iii) Use the identity in part **(ii)** to obtain an expression for $\cos 4\theta$ as a polynomial in $\cos\theta$. **[2]**

(b) **(i)** Given that $z = 4\mathrm{e}^{\mathrm{j}\pi/3}$ and that $w^2 = z$, write down the possible values of w in the form $r\mathrm{e}^{\mathrm{j}\theta}$, where $r > 0$. Show z and the possible values of w in an Argand diagram. **[5]**

(ii) Find the least positive integer n for which z^n is real.

Show that there is no positive integer n for which z^n is imaginary.

For each possible value of w, find the value of w^3 in the form $a + \mathrm{j}b$ where a and b are real. **[5]**

3 **(i)** Find the value of a for which the matrix

$$\mathbf{M} = \begin{pmatrix} 1 & 2 & 3 \\ -1 & a & 4 \\ 3 & -2 & 2 \end{pmatrix}$$

does not have an inverse.

Assuming that a does not have this value, find the inverse of $\mathbf{M}$ in terms of a. **[7]**

(ii) Hence solve the following system of equations.

$$\begin{aligned} x + 2y + 3z &= 1 \\ -x \qquad + 4z &= -2 \\ 3x - 2y + 2z &= 1 \end{aligned}$$ **[4]**

(iii) Find the value of b for which the following system of equations has a solution.

$$\begin{aligned} x + 2y + 3z &= 1 \\ -x + 6y + 4z &= -2 \\ 3x - 2y + 2z &= b \end{aligned}$$

Find the general solution in this case and describe the solution geometrically. **[7]**

Section B (18 marks)

Answer one question

Option 1: Hyperbolic functions

4 **(i)** Prove, from definitions involving exponential functions, that

$$\cosh 2u = 2\sinh^2 u + 1. \quad \textbf{[3]}$$

(ii) Prove that, if $y \geqslant 0$ and $\cosh y = u$, then $y = \ln\left(u + \sqrt{(u^2 - 1)}\right)$. **[4]**

(iii) Using the substitution $2x = \cosh u$, show that

$$\int \sqrt{4x^2 - 1}\,\mathrm{d}x = ax\sqrt{4x^2 - 1} - b\,\mathrm{arcosh}\,2x + c,$$

where a and b are constants to be determined and c is an arbitrary constant. **[7]**

(iv) Find $\int_{\frac{1}{2}}^{1} \sqrt{4x^2 - 1}\,\mathrm{d}x$, expressing your answer in an exact form involving logarithms. **[4]**

Option 2: Investigation of curves

This question requires the use of a graphical calculator.

5 This question concerns curves with polar equation $r = \sec\theta + a$, where a is a constant.

(i) State the set of values of θ between 0 and 2π for which r is undefined. **[2]**

For the rest of the question you should assume that θ takes all values between 0 and 2π for which r is defined.

(ii) Use your graphical calculator to obtain a sketch of the curve in the case $a = 0$. Confirm the shape of the curve by writing the equation in cartesian form. **[3]**

(iii) Sketch the curve in the case $a = 1$.

Now consider the curve in the case $a = -1$. What do you notice?

By considering both curves for $0 < \theta < \pi$ and $\pi < \theta < 2\pi$ separately, describe the relationship between the cases $a = 1$ and $a = -1$. **[6]**

(iv) What feature does the curve exhibit for values of a greater than 1?

Sketch a typical case. **[3]**

(v) Show that a cartesian equation of the curve $r = \sec\theta + a$ is $(x^2 + y^2)(x - 1)^2 = a^2x^2$. **[4]**

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4756 Further Methods for Advanced Mathematics (FP2)

PRINTED ANSWER BOOK

Candidates answer on this Printed Answer Book.

OCR supplied materials:
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- MEI Examination Formulae and Tables (MF2)

Other materials required:
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Duration: 1 hour 30 minutes

Candidate forename		Candidate surname	

Centre number						Candidate number				

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Section A (54 marks)

1 (a) (i)	
1(a)(ii)(*A*)	

1(a)(ii)(*B*)

1 (b)	

2 (a) (i)	
2 (a) (ii)	

2 (a) (iii)	
2 (b) (i)	

2 (b) (ii)

3 (i)

3 (ii)

3 (iii)	

Section B (18 marks)

4 (i)	
4 (ii)	

4 (iii)

4 (iv)	
5 (i)	

5 (ii)	
5 (iii)	(answer space continued on next page)

5 (iii)	(continued)	
5 (iv)		

5 (v)	

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GCE

Mathematics (MEI)

Advanced GCE

Unit **4756:** Further Methods for Advanced Mathematics

Mark Scheme for June 2012

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Annotations

Annotation	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions

a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

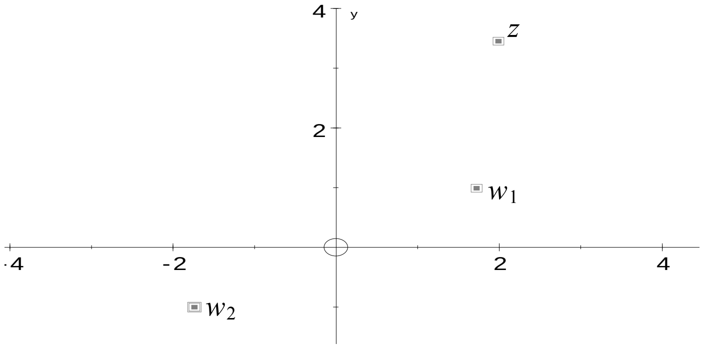
g. Rules for replaced work
If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.
If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.
NB Follow these maths-specific instructions rather than those in the assessor handbook.

h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1	(a)	(i)	$\sin y = x \Rightarrow \cos y \dfrac{dy}{dx} = 1$	M1	Differentiating w.r.t. x or y	$\dfrac{dy}{dx} = \cos y$
			$\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{\cos y}$	A1		
			$\Rightarrow \dfrac{dy}{dx} = (\pm)\dfrac{1}{\sqrt{1-x^2}}$	A1(ag)	Completion www, but independent of B1	$\dfrac{dy}{dx} = \pm\dfrac{1}{\sqrt{1-x^2}}$ or ± not considered scores max. 3
			Taking + sign because gradient is positive	B1	Validly rejecting − sign. Dependent on A1 above	Or $-\dfrac{\pi}{2} \le y \le \dfrac{\pi}{2} \Rightarrow 0 \le \cos y \le 1$
				[4]		
1	(a)	(ii)	(A) $\displaystyle\int_{-1}^{1} \frac{1}{\sqrt{2-x^2}}\,dx = \left[\arcsin\frac{x}{\sqrt{2}}\right]_{-1}^{1}$	M1	arcsin alone, or any appropriate substitution	
				A1	$\arcsin\dfrac{x}{\sqrt{2}}$ or $\int 1\,d\theta$ www	Condone omitted or incorrect limits
			$= \dfrac{\pi}{2}$	A1		
				[3]		
			(B) $\displaystyle\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{\sqrt{1-2x^2}}\,dx = \frac{1}{\sqrt{2}}\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{\sqrt{\frac{1}{2}-x^2}}\,dx$			
				M1	arcsin alone, or any appropriate substitution	
			$= \dfrac{1}{\sqrt{2}}\left[\arcsin\sqrt{2}x\right]_{-\frac{1}{2}}^{\frac{1}{2}}$	A1	$\dfrac{1}{\sqrt{2}}$ and $\sqrt{2}x$ or $\int \dfrac{1}{\sqrt{2}}\,d\theta$ www	
				M1	Using consistent limits in order and evaluating in terms of π. Dependent on M1 above	e.g. $\pm\dfrac{\pi}{4}$ with sub. $x = \dfrac{1}{\sqrt{2}}\sin\theta$
			$= \dfrac{\pi}{2\sqrt{2}}$	A1		
				[4]		

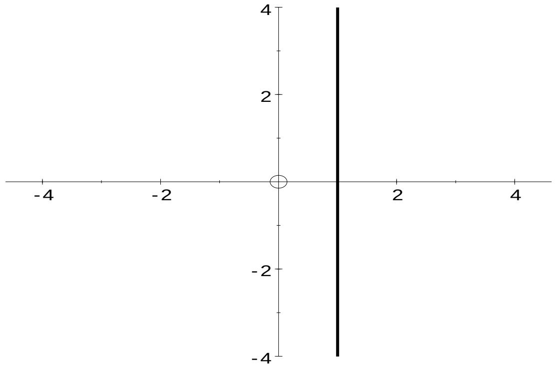
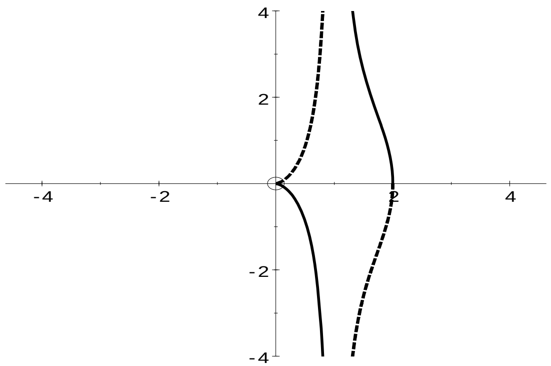
Question			Answer	Marks	Guidance	
1	(b)		$r = \tan\theta$			
			$\Rightarrow\ x = r\cos\theta = \dfrac{\sin\theta}{\cos\theta} \times \cos\theta = \sin\theta$	M1 A1(ag)	Using $x = r\cos\theta$ o.e.	
			$\Rightarrow\ r^2 = \dfrac{\sin^2\theta}{\cos^2\theta} = \dfrac{\sin^2\theta}{1-\sin^2\theta} = \dfrac{x^2}{1-x^2}$	M1 A1(ag)	Obtaining r^2 in terms of x	
			$r^2 = x^2 + y^2 \Rightarrow x^2 + y^2 = \dfrac{x^2}{1-x^2}$			
			$\Rightarrow\ y^2 = \dfrac{x^2}{1-x^2} - x^2$	M1	Obtaining y^2 in terms of x	
			$\Rightarrow\ y^2 = \dfrac{x^2 - x^2(1-x^2)}{1-x^2} = \dfrac{x^4}{1-x^2}$			
			$\Rightarrow\ y = \dfrac{x^2}{\sqrt{1-x^2}}$	A1(ag)		Ignore discussion of ±
			Asymptote $x = 1$	B1 **[7]**	Condone $x = \pm 1$	$x \neq 1$, $x^2 = 1$ B0
2	(a)	(i)	$z^n + \dfrac{1}{z^n} = 2\cos n\theta$	B1	Mark final answer	
			$z^n - \dfrac{1}{z^n} = 2\mathrm{j}\sin n\theta$	B1 **[2]**	Mark final answer	
2	(a)	(ii)	$\left(z + \dfrac{1}{z}\right)^4 = z^4 + 4z^2 + 6 + \dfrac{4}{z^2} + \dfrac{1}{z^4} = z^4 + \dfrac{1}{z^4} + 4\left(z^2 + \dfrac{1}{z^2}\right) + 6$	M1	Expanding by Binomial or complete equivalent	
			$\Rightarrow\ (2\cos\theta)^4 = 2\cos 4\theta + 8\cos 2\theta + 6$	M1 A1	Introducing cosines of multiple angles RHS correct	Condone lost 2s Both As depend on both Ms
			$\Rightarrow\ \cos^4\theta = \dfrac{3}{8} + \dfrac{1}{2}\cos 2\theta + \dfrac{1}{8}\cos 4\theta$	A1ft **[4]**	Dividing both sides by 16. F.t. line above	$A = \dfrac{3}{8}$, $B = \dfrac{1}{2}$, $C = \dfrac{1}{8}$ Give SC2 for fully correct answer found "otherwise"

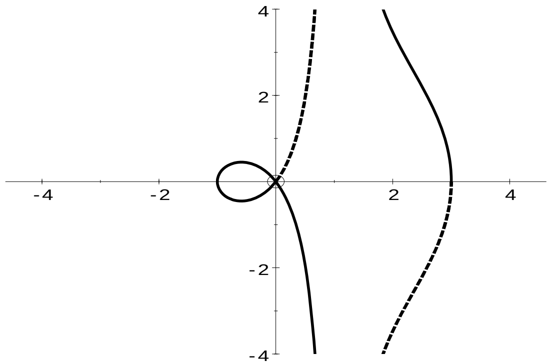
Question			Answer	Marks	Guidance	
2	(a)	(iii)	$\cos^4\theta = \frac{3}{8}+\frac{1}{2}\left(2\cos^2\theta-1\right)+\frac{1}{8}\cos 4\theta$ $\Rightarrow\ \cos^4\theta = \cos^2\theta - \frac{1}{8}+\frac{1}{8}\cos 4\theta$ $\Rightarrow\ \cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$	M1 A1 **[2]**	Using (ii), obtaining $\cos 4\theta$ and expressing $\cos 2\theta$ in terms of $\cos^2\theta$ c.a.o.	Condone $\cos 2\theta = \pm 1 \pm 2\cos^2\theta$
2	(b)	(i)	$z = 4e^{\frac{j\pi}{3}}$ and $w^2 = z$: let $w = re^{j\theta} \Rightarrow w^2 = r^2e^{2j\theta}$ $\Rightarrow\ r^2 = 4 \Rightarrow r = 2$ and $\theta = \frac{\pi}{6}, \frac{7\pi}{6}$ 	 B1 B1B1 B1 B1 **[5]**	 Or $-\frac{5\pi}{6}$ Roots with approx. equal moduli and approx. correct argument Dependent on first B1 z in correct position	 Condone $r = \pm 2$ Award B2 for $\pi\left(k+\frac{1}{6}\right)$ Ignore annotations and scales $\le \pi/4$ Modulus and argument bigger
2	(b)	(ii)	$z = 4e^{\frac{j\pi}{3}} \Rightarrow z^n = 4^n e^{\frac{j\pi n}{3}}$ so real if $\frac{\pi n}{3} = \pi \Rightarrow n = 3$ Imaginary if $\frac{\pi n}{3} = \frac{\pi}{2} + k\pi \Rightarrow n = \frac{3}{2} + 3k$ which is not an integer for any k $w_1 = 2e^{\frac{j\pi}{6}} \Rightarrow w_1{}^3 = 8e^{\frac{j\pi}{2}} = 8j$ $w_2 = 2e^{\frac{7j\pi}{6}} \Rightarrow w_2{}^3 = 8e^{\frac{7j\pi}{2}} = -8j$	B1 M1 A1(ag) M1 A1 **[5]**	 $\cos\frac{\pi n}{3} = 0$ or $\frac{\pi n}{3} = \frac{\pi}{2}$... An argument which covers the positive and negative im. axis Attempting their w^3 in any form 8j, −8j	Ignore other larger values Must deal with mod and arg

Question			Answer	Marks	Guidance	
3	(i)		det ($\mathbf{M}$) = 1(2a + 8) − 2(−2 − 12) + 3(2 − 3a) = 42 − 7a $\Rightarrow$ no inverse if a = 6 $\mathbf{M}^{-1} = \frac{1}{42-7a}\begin{pmatrix} 2a+8 & -10 & 8-3a \\ 14 & -7 & -7 \\ 2-3a & 8 & a+2 \end{pmatrix}$	M1A1 A1 M1 A1 M1 A1 **[7]**	Obtaining det($\mathbf{M}$) in terms of a At least 4 cofactors correct (including one involving a) Six signed cofactors correct Transposing and ÷ by det($\mathbf{M}$). Dependent on previous M1M1	Accept unsimplified Accept $a \neq 6$ after correct det M0 if more than 1 is multiplied by the corresponding element Mark final answer
3	(ii)		$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{42}\begin{pmatrix} 8 & -10 & 8 \\ 14 & -7 & -7 \\ 2 & 8 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ $\Rightarrow x = \frac{6}{7}, y = \frac{1}{2}, z = -\frac{2}{7}$	M1 M1 A2 **[4]**	Substituting a = 0 Correct use of inverse Dependent on both M marks. Give A1 for one correct SC1 for x = 6, y = 3.5, z = −2	One correct element. Condone missing determinant After M0, give SC2 for correct solution and SC1 for one correct Answers unsupported score 0
3	(iii)		e.g. 7x − 10y = 10, 7x − 10y = 3b − 2 (or e.g. 4x + 5z = 5, 4x + 5z = b + 1) (or e.g. 8y + 7z = −1, 8y + 7z = 3 − b) For solutions, 10 = 3b − 2 $\Rightarrow$ b = 4	M1 A1 M1 A1	Eliminating one variable in two different ways Two correct equations Validly obtaining a value of b	Or 7x − 10y = 2b + 2 Or 8x + 10z = 3b − 2 Or 16y + 14z = b − 6
			OR M2 A1 b = 4 A1		A method leading to an equation from which b could be found A correct equation	E.g. setting z = 0, augmented matrix, adjoint matrix, etc.
			$x = \lambda, y = 0.7\lambda - 1, z = 1 - 0.8\lambda$ Straight line	M1 A1 B1 **[7]**	Obtaining general soln. by e.g. setting one unknown = λ and finding other two in terms of λ Any correct form Accept "sheaf", "pages of a book", etc.	Accept unknown instead of λ $x = \frac{10}{7}\lambda + \frac{10}{7}, y = \lambda, z = -\frac{8}{7}\lambda - \frac{1}{7}$ $x = \frac{5}{4} - \frac{5}{4}\lambda, y = -\frac{7}{8}\lambda - \frac{1}{8}, z = \lambda$ Independent of all previous marks. Ignore other comments

Question			Answer	Marks	Guidance	
4	(i)		$\sinh u = \dfrac{e^u - e^{-u}}{2} \Rightarrow \sinh^2 u = \dfrac{e^{2u} - 2 + e^{-2u}}{4}$	B1	$\left(e^u - e^{-u}\right)^2 = e^{2u} - 2 + e^{-2u}$	Accept other or mixed variables
			$\Rightarrow 2\sinh^2 u + 1 = \dfrac{e^{2u} - 2 + e^{-2u}}{2} + 1 = \dfrac{e^{2u} + e^{-2u}}{2}$	B1	$\cosh 2u = \dfrac{e^{2u} + e^{-2u}}{2}$	
			$= \cosh 2u$	B1	Completion www	
				[3]		
4	(ii)		If $\cosh y = u$, $u = \dfrac{e^y + e^{-y}}{2}$	M1	Expressing u in exponential form	$\frac{1}{2}$, + must be correct
			$\Rightarrow e^y + e^{-y} = 2u \Rightarrow e^{2y} - 2ue^y + 1 = 0$			
			$\Rightarrow \left(e^y - u\right)^2 - u^2 + 1 = 0$			
			$\Rightarrow e^y = u \pm \sqrt{u^2 - 1}$	M1	Reaching e^y	Condone omitted ±
			$\Rightarrow y = \ln\left(u + \sqrt{u^2 - 1}\right)$	A1(ag)	Completion www; indep. of B1	$y = \ln\left(u \pm \sqrt{u^2 - 1}\right)$ or ± not considered scores max. 3
			$y \ge 0 \Rightarrow e^y = u + \sqrt{u^2 - 1}$	B1	Validly rejecting − sign Dependent on A1 above	
			OR $\ln\left(u + \sqrt{u^2 - 1}\right) = \ln\left(\cosh y + \sqrt{\cosh^2 y - 1}\right)$ M1		Substituting $u = \cosh y$	
			$= \ln\left(\cosh y + \sinh y\right)$			
			since $\sinh y > 0$ B1		Rejecting −ve square root	Dependent on A1
			$= \ln\left(e^y\right)$ M1		Reaching e^y	
			$= y$ A1		Completion www; indep. of B1	
				[4]		

Question			Answer	Marks	Guidance	
4	(iii)		$x=\frac{1}{2}\cosh u \Rightarrow \frac{dx}{du}=\frac{1}{2}\sinh u$	M1	Reaching integrand equivalent to $k\sinh^2 u$	
			$\int\sqrt{4x^2-1}\,dx=\int\sqrt{\cosh^2 u-1}\times\frac{1}{2}\sinh u\,du$			
			$=\int\frac{1}{2}\sinh^2 u\,du$	A1		
			$=\int\frac{1}{4}\cosh 2u-\frac{1}{4}\,du$	M1	Simplifying to integrable form. Dependent on M1 above	Or $\frac{1}{8}e^{2u}-\frac{1}{4}+\frac{1}{8}e^{-2u}$
			$=\frac{1}{8}\sinh 2u-\frac{1}{4}u+c$	A1A1	For $\frac{1}{8}\sinh 2u$ o.e. and $-\frac{1}{4}u$ seen	Or $\frac{1}{16}e^{2u}-\frac{1}{4}u-\frac{1}{16}e^{-2u}+c$
			$=\frac{1}{4}\sinh u\cosh u-\frac{1}{4}u+c$			Condone omission of $+\,c$ throughout
			$=\frac{1}{4}\sqrt{4x^2-1}\times 2x-\frac{1}{4}\operatorname{arcosh}2x+c$	M1	Clear use of $\sinh 2u = 2\sinh u\cosh u$ Dependent on M1M1 above	
			$=\frac{1}{2}x\sqrt{4x^2-1}-\frac{1}{4}\operatorname{arcosh}2x+c$			
			$a=\frac{1}{2}$	A1		a, b need not be written separately
				[7]		
4	(iv)		$\int_{\frac{1}{2}}^{1}\sqrt{4x^2-1}\,dx=\left[\frac{1}{2}x\sqrt{4x^2-1}-\frac{1}{4}\operatorname{arcosh}2x\right]_{\frac{1}{2}}^{1}$	M1	Using their (iii) and using limits correctly	
			$=\frac{\sqrt{3}}{2}-\frac{1}{4}\operatorname{arcosh}2+\frac{1}{4}\operatorname{arcosh}1$	A1ft	May be implied F.t. values of a and b in (iii)	$a\sqrt{3}-b\operatorname{arcosh}2$. No decimals. Must have obtained values for a and b
			$=\frac{\sqrt{3}}{2}-\frac{1}{4}\ln\left(2+\sqrt{3}\right)+\frac{1}{4}\ln 1$	M1	Using (ii) accurately Dependent on M1 above	
			$=\frac{\sqrt{3}}{2}-\frac{1}{4}\ln\left(2+\sqrt{3}\right)$	A1	c.a.o. A0 if ln 1 retained Mark final answer	Correct answer www scores 4/4
				[4]		

Question			Answer	Marks	Guidance	
5	(i)		Undefined for $\theta = \frac{\pi}{2}$ and $\frac{3\pi}{2}$	B1B1 **[2]**		
5	(ii)		 $r = \sec\theta \Rightarrow r\cos\theta = 1$ $\Rightarrow x = 1$	B1 M1 A1 **[3]**	Vertical line through (1, 0) (indicated, e.g. by scale) Use of $x = r\cos\theta$	
5	(iii)		$a = 1$:  $a = -1$ gives same curve $a = 1$, $0 < \theta < \pi$ corresponds to $a = -1$, $\pi < \theta < 2\pi$ $a = -1$, $0 < \theta < \pi$ corresponds to $a = 1$, $\pi < \theta < 2\pi$	B1 B2 B1 B1 B1 **[6]**	Section through (2, 0) (indicated) Section through (0, 0) (give B1 for one error)	If asymptote included max. 2/3

Question			Answer	Marks	Guidance	
5	(iv)		Loop e.g. $a = 2$ 	B1 B2 **[3]**	 Give B1 for one error	
5	(v)		$r = \sec\theta + a$ $\Rightarrow \; r = \dfrac{r}{x} + a$ $\Rightarrow \; r\left(1 - \dfrac{1}{x}\right) = a$ $\Rightarrow \; \sqrt{x^2 + y^2}\left(\dfrac{x-1}{x}\right) = a$ $\Rightarrow \; \sqrt{x^2 + y^2}\,(x-1) = ax$ $\Rightarrow \; \left(x^2 + y^2\right)(x-1)^2 = a^2x^2$	 M1 M1 M1 A1(ag) **[4]**	 Use of $x = r\cos\theta$ Use of $r = \sqrt{x^2 + y^2}$ Correct manipulation	

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4756 Further Methods for Advanced Mathematics (FP2)

General Comments

Most candidates found this paper accessible and were able to provide evidence of what they knew, understood and could do across the whole specification. More than one-third of candidates scored at least 60 marks and only about 5% scored 20 marks or fewer. Question 1, on calculus and polar co-ordinates, produced the highest scores, while Question 4 on hyperbolic functions yielded the lowest mean. Fewer than 1% of candidates attempted Question 5 on investigations of curves: this option is to be examined for the last time in this paper in January 2013.

Candidates appeared well-versed in the standard results and processes which appear at this level, for example Q1(a)(ii) on arcsin integrals, Q2(b)(i) on square roots of a complex number, Q3(i) on the inverse of a 3 × 3 matrix and Q4(i) on a hyperbolic identity. Most proofs and methods offered in these questions were clear and concise. As might be expected, the part-questions yielding the lowest average scores were those which covered slightly less familiar ground, such as Q1(b), Q2(b)(ii), Q3(iii) and Q4(iii) and (iv). Although much fluency was seen, even very competent candidates could be seen struggling to keep control of their signs, for example in Q3(iii).

The structure of questions is intended to assist candidates. Thus it was surprising to find some candidates using inverse hyperbolic functions in the integrals in Q1(a), in which they have already been asked to differentiate arcsin; ignoring the instruction to “use the identity in part (ii)” in Q2(a)(iii); not using “hence” in Q3(ii); and not spotting the connection between the indefinite and the definite integrals in Q4(iii) and (iv).

Comments on Individual Questions

1) **Calculus with inverse trigonometric functions; polar co-ordinates**

(a) (i) The first three marks were obtained quickly and easily by most candidates. Very few scored the final mark by giving an explanation as to why the positive square root is chosen. Most did not seem to realise that there was a choice. A very few just asserted the result “from the formula book”.

(ii) Almost all candidates realised that these were inverse sine integrals. (A) was very well done with about 90% of candidates scoring full marks. In (B), a few more errors with the constants crept in. Most used the “standard result” rather than attempting a substitution from first principles. A very few tried to use inverse hyperbolic functions, and another small group insisted on using degrees.

(b) The best answers were concise and clearly established each answer, leaving no details to be filled in by the examiner. Proving that $x = \sin\theta$ was probably the trickiest part. The other two expressions were derived more consistently and the equation of the asymptote was frequently correct.

2) **Complex numbers**

(a) (i) Most candidates achieved full marks here. The most common slips involved missing out the *j*s or the *n*s.

(ii) Most candidates knew what they had to do and the binomial expansion was usually carried out correctly. The main error was to omit the factors of 2 when introducing trigonometric functions on both sides of the expansion. A small number tried to use a succession of trigonometric identities to solve the problem, despite the instruction in the question to begin with $(z + 1/z)^4$.

(iii) Most realised that they had to substitute for cos 2θ but could not do so accurately, with a large number thinking that $\cos 2\theta = \cos^2\theta - 1$. Poor algebraic manipulation prevented many from achieving the correct answer.

(b) (i) This part was done very well, with the best answers achieving full marks with just a few lines of working and a clear diagram. Some ignored the instruction to give $r > 0$. A few squared z rather than finding its square roots, and a few had more than two square roots.

(ii) The first mark, for $n = 3$, was frequently scored, although many thought 0 was a positive integer (some scripts contained debate about whether 0 was an integer at all). Showing that z^n was never imaginary was found quite difficult. A whole variety of different explanations was seen: the best answers displayed clear thinking, were easy to follow, and covered both the positive and negative imaginary axes. Many candidates thought it was enough to assert that $n\pi/3 = \pi/2 \Rightarrow n = 3/2$ which was not an integer, while others thought that, if z^n were imaginary, $\sin n\pi/3 = 1$. Some candidates produced extended essays. The final two marks were easier to obtain and even if candidates had incorrect values of w they usually scored the method mark for cubing them. A very few obtained w in the form $a + jb$ and tried to cube that.

3) **Matrices**

(i) The majority of candidates achieved full marks in this part. Most were able to find the determinant accurately, most often by expanding by the first row although Sarrus' method was also quite popular. As always, there were a few sign errors. A minority of candidates, having obtained $42 - 7a$, used $6 - a$ as the determinant, which also affected part (ii): this was dealt with by a special case in the mark scheme. As is usual, the inverse matrix was found efficiently and accurately. Errors, when they appeared, included: failing to change the signs of the minors to obtain the cofactors; sign errors in the cofactors; forgetting to transpose; and multiplying the cofactors by the corresponding elements.

(ii) Again, this was well done. A minority of candidates ignored the "hence" in the question and used an algebraic method to solve the equations: this scored a maximum of 2/4. A very few quoted the answer without working, presumably from a calculator: this did not receive any credit.

(iii) The best solutions, once again, were concise and easy to follow, often producing the required answer in a few lines. The "standard" approach was to eliminate the same unknown between two different pairs of equations, and poor manipulative algebra often prevented competent candidates from obtaining the correct value of b: they would be well advised not to try to do so much in their heads! "Line" or "sheaf" (or even "sheath") appeared fairly often, while finding an accurate general solution proved challenging and, here again, poor manipulation was the main barrier to full marks. Some found a point and then stated the solution was a line.

4) **Hyperbolic functions**

(i) This was done well with over three-quarters of candidates obtaining full marks. Again, the best proofs were concise and clear, leaving no details for the examiner to fill in. Other candidates lost all three marks by ignoring the instruction in the question to "prove from definitions involving exponential functions".

(ii) This was also done well and it was pleasing that so many candidates considered the $\pm$ and gave valid reasons why the $-$ sign should be dropped. Others gave spurious reasons such as "you cannot take the natural logarithm of a negative number".

(iii) Most candidates used the suggested substitution accurately although some candidates found it hard to find *dx* in terms of *du* and did not score. Then most used a hyperbolic identity to obtain an integrable form, while a small minority converted everything to exponentials. Obtaining the last two marks, by putting the result of the integration into the required form, was found challenging, and the methods used were not always transparent. Some attempted to express $\sinh 2u$ in terms of exponentials, but very few were fully successful by this method.

(iv) Many did not attempt this part or, even after having attempted (iii), started again, not realising that all that was expected was substitution of the limits. Those who made progress usually tried to give the answer in the required "exact form" and used part (ii) accurately.

5) **Investigations of curves**

Very few candidates attempted this question and, although there were a few good attempts, the evidence suggests that candidates tried this question mainly after an unsuccessful attempt at Q4.

GCE Mathematics (MEI)

			Max Mark	90% cp	a	b	c	d	e	u
4753/01	(C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	66	60	53	47	41	34	0
4753/02	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	16	15	13	11	9	8	0
4753/82	(C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	16	15	13	11	9	8	0
4753	(C3) MEI Methods for Advanced Mathematics with Coursework	UMS	100	90	80	70	60	50	40	0
4754/01	(C4) MEI Applications of Advanced Mathematics	Raw	90	73	65	57	50	43	36	0
		UMS	100	90	80	70	60	50	40	0
4756/01	(FP2) MEI Further Methods for Advanced Mathematics	Raw	72	66	61	53	46	39	32	0
		UMS	100	90	80	70	60	50	40	0
4757/01	(FP3) MEI Further Applications of Advanced Mathematics	Raw	72	61	54	47	40	34	28	0
		UMS	100	90	80	70	60	50	40	0
4758/01	(DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	68	63	57	51	45	39	0
4758/02	(DE) MEI Differential Equations with Coursework: Coursework	Raw	18	16	15	13	11	9	8	0
4758/82	(DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	16	15	13	11	9	8	0
4758	(DE) MEI Differential Equations with Coursework	UMS	100	90	80	70	60	50	40	0
4762/01	(M2) MEI Mechanics 2	Raw	72	65	58	51	44	38	32	0
		UMS	100	90	80	70	60	50	40	0
4763/01	(M3) MEI Mechanics 3	Raw	72	67	63	56	50	44	38	0
		UMS	100	90	80	70	60	50	40	0
4764/01	(M4) MEI Mechanics 4	Raw	72	63	56	49	42	35	29	0
		UMS	100	90	80	70	60	50	40	0
4767/01	(S2) MEI Statistics 2	Raw	72	66	61	55	49	43	38	0
		UMS	100	90	80	70	60	50	40	0
4768/01	(S3) MEI Statistics 3	Raw	72	65	58	51	44	38	32	0
		UMS	100	90	80	70	60	50	40	0
4769/01	(S4) MEI Statistics 4	Raw	72	63	56	49	42	35	28	0
		UMS	100	90	80	70	60	50	40	0
4772/01	(D2) MEI Decision Mathematics 2	Raw	72	62	56	50	44	39	34	0
		UMS	100	90	80	70	60	50	40	0
4773/01	(DC) MEI Decision Mathematics Computation	Raw	72	52	46	40	34	29	24	0
		UMS	100	90	80	70	60	50	40	0
4777/01	(NC) MEI Numerical Computation	Raw	72	63	55	47	39	32	25	0
		UMS	100	90	80	70	60	50	40	0